

HEALTH AND WEALTH IN A DYNAMIC GENERAL EQUILIBRIUM THEORY

Wei-Bin ZHANG

Ritusmeikan Asia Pacific University, Japan

wbz1@apu.ac.jp

Abstract

This paper deals with dynamic interdependence between wealth and health in a dynamic general equilibrium theory. The economy is composed of any number of groups of people. It consists of three economic sectors - capital good, consumer good, and health sectors. We describe the economic structure and production technologies on the basis of Walrasian general equilibrium theory and neoclassical growth theory. Zhang's utility function is applied to describe behavior of households. In our approach wealth and income inequalities between households are caused by heterogeneity in households' preferences and differences in characteristics of health and human accumulation. Markets are perfectly competitive. Wealth accumulation is through saving and change in health stock is through health caring and consuming goods and services. We first build the dynamic general equilibrium model and then we provide a computational procedure so that we can easily follow movement of the economic system with specified parameter values and proper initial conditions. We simulate the heterogeneous-household model with three types of households. We identify the existence of a locally stable equilibrium point for the given parameters. We plot the motion of the economy and carry out comparative dynamic analysis with regard to changes in some parameters.

Keywords: *health, wealth accumulation, Walrasian general equilibrium theory; neoclassical growth theory; economic structure*

I. INTRODUCTION

Health is a well-recognized important determinant of economic development, even though most economic growth models do not give an explicit treatment of this behavior-dependent variable. Although most of the literature in economic growth theory treats physical capital as the single endogenous determinant of economic growth, it has become clear that it is essential to include some other endogenous determinants such as human capital, environment, health, and preference changes in order to explain dynamic processes of economic development. The purpose of this study is to make a unique contribution to the literature of economic growth theory with endogenous health by extending and synthesizing Walrasian general economic theory and neoclassical growth theory with Zhang's utility function to explore dynamic interdependence between physical wealth and health accumulation.

It is obvious that there are different economic mechanisms and processes of health and wealth accumulation. Health caring consumes incomes, while saving implies delaying consumption to the future. Healthier workers tend to have higher productivity and thus tend to get higher wage income and increase saving, while saving accumulates wealth and delays current consumption and thus may reduce health if lower consumption does not provide sufficient nutrition. Worker's productivity is enhanced by higher physical capacities, such as strength and endurance. Healthier workers are obviously physically and mentally more energetic and robust. We see that there are close interactions between health and wealth and these interactions are dependent on households' preferences, technologies, and economic structures. This study deals with dynamics of physical wealth and health on the basis of the Walrasian general equilibrium theory of pure exchange and production economies. Walrasian theory was initiated by Walras and further refined, generalized, and extended by Arrow, Debreu and others mainly in the 1950s (e.g., Walras, 1874; Arrow and Debreu, 1954; Gale, 1955; Nikaido, 1956, 1968; Debreu, 1959; McKenzie, 1959; Arrow and Hahn, 1971; Arrow, 1974; and Mas-Colell et al., 1995). It mainly studies market equilibrium with economic mechanisms of production, consumption, and exchanges with heterogeneous industries and households. It is essentially static as it does not include endogenous changes of, for instance, physical capital, human capital, and environment. Our model is Walrasian in the sense that for given levels of wealth and health there are competitive market equilibria with heterogeneous industries and households. Although much effort has been done to include endogenous wealth in Walrasian theory (e.g., Morishima, 1964, 1977; Diewert, 1977; Eatwell, 1987; Dana et al. 1989; Jensen and Larsen, 2005; Montesano, 2008; Impicciatore et al., 2012), there are few models which take account of endogenous health changes.

As far as capital accumulation is concerned, neoclassical growth theory is the key tool for economists to explain growth with capital as a main determinant (e.g., Solow, 1956; Uzawa, 1961, 1963; Stiglitz, 1967; Burmeister and Dobell, 1970; and Barro and Sala-i-Martin, 1995). This study integrates neoclassical growth theory and Walrasian general theory to deal with a heterogeneous-household and multi-sector economy with

endogenous wealth and health. A main deviation from mainstreams of economics is related to modelling behaviour of households. We apply Zhang's utility function to model human behavior (Zhang, 1993, 2005). Much effort has been made to theoretically or empirically examine possible interactions between health and economic systems (e.g., Parkin et al. , 1987; Posnett and Hitiris, 1992; Strauss and Thomas, 1998; Rivera and Currais, 1999; Schultz, 1999; Bloom, et al., 2004; Fletcher, 2012; Klaus, et al. 2013; Fletcher and Frisvold, 2014; and Pestieau and Racionero, 2016). As far as modelling health change and relation between health and productivity are concerned, this study is based on Grossman (1972), van Zon and Muysken (2001), Kelly (2017) and Zhang (2018). The main contribution of this paper is to develop the ideas about health and economic growth into a dynamic general equilibrium framework.

It synthesizes two papers recently proposed by Zhang (2012, 2018). Zhang (2012) builds a dynamic general equilibrium model by integrating Walrasian theory and neoclassical growth theory with endogenous capital but without health. Zhang (2018) introduces endogenous health to traditional neoclassical growth theory. This paper integrates the main ideas in the two models to deal with interdependence between economic structure, income and wealth distributions, wealth accumulation, and health change. The rest of the paper is organized as follows. Section 2 defines the heterogeneous-household and multi-sector growth model with endogenous health and wealth. Section 3 shows how we solve the dynamics of the heterogeneous-household and multi-sector and simulates the motion of the economic system. Section 4 studies comparative dynamic analysis with regard to different exogenous changes. Section 5 makes conclusions of the study. The appendix proves the main results in Section 3.

II. THE MULTI-COUNTRY GROWTH MODEL WITH HEALTH CARING

This paper constructs a general equilibrium dynamic growth model of endogenous wealth and health accumulation with heterogeneous households. The economy produces capital good and consumer good and supplies health service. We apply neoclassical growth theory to describe capital good sector, consumer good sector, and health sector. Most aspects of the three sectors are the same as in production sectors in neoclassical growth theory (Solow, 1956; and Zhang, 2005). Households own all assets. Households spend their disposable incomes on consuming two goods, saving, and health caring. Production sectors employ capital and labor as input factors with constant technologies. Markets are perfectly competitive. Factor markets work well and available input factors are fully utilized. The population is classified into J groups. We introduce

- j - index standing for group j , $j = 1, \dots, J$;
- \bar{N}_j and $N_j(t)$ - fixed population and total labor supply of group j , $j = 1, \dots, J$;
- $\ell_j(t)$ - level of health stock of group j , $j = 1, \dots, J$;
- i , s and h - subscript index standing for capital good sector, consumer good sector, and health sector;
- $N_q(t)$ and $K_q(t)$ - labor force and capital stocks employed by sector q , $q = i, s, h$;
- $F_q(t)$ - output level of sector q ;
- $T_j(t)$, $\bar{T}_j(t)$, and $\hat{T}_j(t)$ - household j 's work time, leisure time, and time spent on health caring;
- $c_j(t)$, $c_{js}(t)$ and $c_{jh}(t)$ - household j 's consumption levels of capital good, consumer good and health service;
- $\bar{k}_j(t)$ - household j 's wealth, $j = 1, \dots, J$;
- $p_s(t)$ and $p_h(t)$ - prices of consumer good and health service;
- $r(t)$ and $w(t)$ - rate of interest in global markets and wage rate; and
- δ_k and δ_{jh} - constant depreciation rates of physical capital and household j 's health stock.

The labor supply

Total labor supply is a function of populations, human capital, health, and work hours. We specify the following national labor supply

$$N(t) = \sum_{j=1}^J h_j T_j(t) \ell_j^{m_j}(t) \bar{N}_j, \quad (1)$$

where h_j is fixed level human capital of group j , and m_j measures how worker's health affects labor's productivity (Weil, 2007; Tobing, 2011; and Zhang, 2018).

Production functions of the three sectors

We describe production functions of the three sectors with the Cobb-Douglas forms

$$F_q(t) = A_q K_q^{\alpha_q}(t) N_q^{\beta_q}(t), \quad A_q, \alpha_q, \beta_q > 0, \quad \alpha_q + \beta_q = 1, \quad (2)$$

where A_q , α_q , and β_q are positive parameters.

The marginal conditions

Profit maximization in perfectly competitive markets implies the following marginal conditions for the three sectors

$$r(t) + \delta_k = \frac{\alpha_q p_q(t) F_q(t)}{K_q(t)}, \quad w(t) = \frac{\beta_q p_q(t) F_q(t)}{N_q(t)}, \quad q = i, s, h, \quad (3)$$

where $p_i(t) = 1$.

Health caring as a function of health service and time spent on health caring

Grossman (1972) describes personal production function with time input as follows: "Consumers produce commodities with inputs of market goods and their own time. For example, they use traveling time and transportation services to produce visits; part of their Sundays and church services to produce 'peace of mind'; and their own time, books, and teachers' services to produce additions to knowledge." With the same spirit, we may consider health caring as a joint production of health service $c_{jh}(t)$ and time spent on health caring $\hat{T}_j(t)$. We use $\bar{c}_j(t)$ to stand for output of health caring. We take on the following form of health caring function

$$\bar{c}_j(t) = \bar{A}_j c_{jh}^{\bar{\alpha}_j}(t) \hat{T}_j^{\bar{\beta}_j}(t), \quad \bar{A}_j, \bar{\alpha}_j, \bar{\beta}_j > 0, \quad (4)$$

where \bar{A}_j , $\bar{\alpha}_j$, and $\bar{\beta}_j$ are parameters.

Household behaviors

This study applies Zhang's approach to household behavior (Zhang, 1993, 2005). There are five variables for households to decide. They are leisure time, health caring, and consumption of capital good and consumer good, and saving. Household j 's current income from the interest payment $r(t)\bar{k}_j(t)$ and the wage income $W_j(t)$ is

$$y_j(t) = r(t)\bar{k}_j(t) + W_j(t), \quad (5)$$

where

$$W_j(t) \equiv h_j \ell_j^{m_j}(t) T_j(t) w(t).$$

It should be noted that the disposable income in contemporary macroeconomics is the current income in Zhang's approach. In Zhang's approach, the disposable income is the sum of the current income and the value of wealth. The disposable income is

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t) = R(t)\bar{k}_j(t) + W_j(t), \quad (6)$$

where $R(t) \equiv 1 + r(t)$. The disposable income is distributed between health caring, and consumption of capital good and consumer good, and saving. The budget constraint implies

$$c_j(t) + p_s(t)c_{js}(t) + p_h(t)c_{jh}(t) + s_j(t) = \hat{y}_j(t). \quad (7)$$

The consumer's time constraint implies

$$T_j(t) + \bar{T}_j(t) + \hat{T}_j(t) = T_0, \quad (8)$$

where T_0 is the total available time equally given for everyone. Insert (6) and (8) in (7)

$$\bar{p}_j(t)\bar{T}_j(t) + \bar{p}_j(t)\hat{T}_j(t) + c_j(t) + p_s(t)c_{js}(t) + p_h(t)c_{jh}(t) + s_j(t) = \bar{y}_j(t), \quad (9)$$

in which

$$\bar{p}_j(t) \equiv h_j \ell_j^{m_{jh}}(t)w(t).$$

The variable $\bar{p}_j(t)$ is the opportunity cost of health caring and the opportunity cost of leisure time, and $\bar{y}(t)$ implies the potential income that the household can earn by using up the available time on working.

Utility functions and optimal behavior

The household chooses the five variables. Household j 's utility function is taken on the following form

$$U_j(t) = \bar{T}_j^{\sigma_{j0}}(t)c_j^{\xi_{j0}}(t)c_{js}^{\gamma_{j0}}(t)\bar{c}_{jh}^{\psi_{j0}}(t)s^{\lambda_{j0}}(t), \quad \sigma_{j0}, \xi_{j0}, \gamma_{j0}, \psi_{j0}, \lambda_{j0} > 0, \quad (10)$$

where σ_{j0} is called the propensity to consume time, ξ_{j0} the propensity to consume trade goods, γ_{j0} the propensity to consume non-trade goods, ψ_{j0} the propensity to be engaged in health caring, and λ_{j0} the propensity to own wealth. Zhang (2005) applied this utility function to different dynamic problems. According to Grossman (1972), "what consumers demand when they purchase medical services are not these services per se but, rather, 'good health.' Given that the basic demand is for good health, it seems logical to study the demand for medical care by first constructing a model of the demand for health itself." We enter health caring $\bar{c}_{jh}(t)$, rather than health service $c_{jh}(t)$, into the utility function. It should be noted that Newhouse (1977) examines relationships between health care expenditures and income and the magnitude of income elasticity of expenditures. Yavuz *et al.* (2013) identify the income elasticities in different economies. Baltagi and Moscone (2010) empirically conclude that the health expenditure is a necessity good for 20 OECD countries. It is not difficult to see that we can deal with these issues by properly introducing endogenous propensities to consume health caring.

Insert (4) in (10)

$$U_j(t) = \bar{A}_j^{\psi_{j0}} \bar{T}_j^{\sigma_{j0}}(t)\hat{T}_j^{\psi_{j0}\bar{\beta}_j}(t)c_j^{\xi_{j0}}(t)c_{js}^{\gamma_{j0}}(t)c_{jh}^{\psi_{j0}\bar{\alpha}_j}(t)s^{\lambda_{j0}}(t). \quad (11)$$

The first-order conditions for maximize (11) subject to (9) imply

$$\begin{aligned} \bar{p}_j(t)\bar{T}_j(t) &= \sigma_j \bar{y}_j(t), & \bar{p}_j(t)\hat{T}_j(t) &= \hat{\psi}_j \bar{y}_j(t), & c_j(t) &= \xi_j \bar{y}_j(t), \\ p_s(t)c_{js}(t) &= \gamma_j \bar{y}_j(t), & p_h(t)c_{jh}(t) &= \psi_j \bar{y}_j(t), & s_j(t) &= \lambda_j \bar{y}_j(t), \end{aligned} \quad (12)$$

where

$$\sigma_j \equiv \sigma_{j0} \rho_j, \quad \xi_j \equiv \xi_{j0} \rho_j, \quad \gamma_j \equiv \gamma_{j0} \rho_j, \quad \psi_j \equiv \psi_{j0} \bar{\alpha}_j \rho_j, \quad \hat{\psi}_j \equiv \psi_{j0} \bar{\beta}_j \rho_j, \quad \lambda_j \equiv \lambda_{j0} \rho_j,$$

$$\rho_j = \frac{1}{\sigma_{j0} + \xi_{j0} + \gamma_{j0} + \psi_{j0} \bar{\alpha}_j + \psi_{j0} \bar{\beta}_j + \lambda_{j0}}.$$

Wealth accumulation

According to the definition of $s_j(t)$ and $\bar{k}_j(t)$, the change in the household's wealth implies

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{13}$$

The change in wealth is saving minus dissaving.

Changes of health stocks

We now describe changes in health stocks. Dynamics of health stock is related to nutrition, health caring, lifestyle, and health depreciation. As in Zhang (2018), we specify the health dynamics as follows

$$\dot{\ell}_j(t) = \frac{\bar{v}_{jh} \bar{c}^{a_{jh}}(t)}{\ell_j^{\pi_{jh}}(t)} + \frac{\bar{v}_{jc} c_j^{a_{jc}}(t) c_{js}^{a_{js}}(t) \bar{T}_j^{a_{jT}}(t)}{\ell_j^{\pi_{jc}}(t)} - \delta_{jh} \ell_j(t), \tag{14}$$

where $\delta_{jh} (> 0)$ is depreciation rate of health stock, \bar{v}_{jh} , \bar{v}_{jc} , \bar{a}_{jh} , \bar{a}_{jc} and \bar{a}_{jT} are non-negative parameters. We don't specify signs of returns-to-scale parameters $\bar{\pi}_{jh}$ and $\bar{\pi}_{jc}$ as they may be negative (when there are increasing returns to scale) or positive (when there are decreasing returns to scale). It should be remarked that rather than being constant, rate of health depreciation may be related to health stock and other variables. It should be noted that Johansson and Löfgren (1995) describe dynamics of health stock with the following differential equations

$$\dot{\ell}(t) = g(z(t), k_h(t)) - \bar{\delta}_h \ell(t),$$

where $z(t)$ is the level of pollutants and $k_h(t)$ is the capital input for health caring. Grossman (1972) uses a similar differential equation with $g(t)$ related to time input to health caring, the level of human capital and the expenditure on medical care.

Demand and supply

The demand of and supply of consumer balances at any point in time

$$\sum_{j=1}^J c_{js}(t) \bar{N}_j = F_s(t). \tag{15}$$

The demand of and supply of health balances at any point in time

$$\sum_{j=1}^J c_{jh}(t) \bar{N}_j = F_h(t). \tag{16}$$

As the national output of capital good is equal to the sum of the consumption of the good, the depreciation of capital stock and the net savings, we have

$$\sum_{j=1}^J [c_j(t) + s_j(t)] \bar{N}_j = F_i(t) + K(t) - \delta_k K(t). \tag{17}$$

Balances of physical capital

The national capital stock is fully employed

$$K_i(t) + K_s(t) + K_h(t) = K(t). \tag{18}$$

All the national wealth is held by households

$$\sum_{j=1}^J \bar{k}_j(t) \bar{N}_j = K(t). \tag{19}$$

Full employment of the labor force

Assumption of full employment of labor force implies

$$N_i(t) + N_s(t) + N_h(t) = N(t). \tag{20}$$

We constructed the heterogeneous household model with economic structure. All the markets are perfectly competitive. The model is based on some main ideas in economic growth theory, Walrasian general equilibrium theory, and health economics in a comprehensive framework. From a structural point of view, the model is general as some well-known models in economic theory are special cases of the model. We now deal with dynamic properties of the model.

III. THE DYNAMICS OF THE HETEROGENEOUS-HOUSEHOLD MODEL

We now study dynamic properties of the heterogeneous-household general equilibrium model. The following lemma gives a computational procedure to demonstrate the movement of all the variables in the economic system with computer. We introduce

$$z(t) \equiv \frac{r(t) + \delta_k}{w(t)}, \quad \{\bar{k}_j(t)\} \equiv (\bar{k}_2(t), \dots, \bar{k}_J(t)).$$

Lemma

The dynamics of the economic system is given by the following $2J$ differential equations

$$\begin{aligned} \dot{z}(t) &= \Lambda_1(z(t), \{\bar{k}_j(t)\}, (\ell_j(t))), \\ \dot{\bar{k}}_j(t) &= \Lambda_j(z(t), \{\bar{k}_j(t)\}, (\ell_j(t))), \\ \dot{\ell}_j(t) &= \Psi_j(z(t), \{\bar{k}_j(t)\}, (\ell_j(t))), \end{aligned} \tag{21}$$

with $z(t)$, $(\ell_j(t))$, and $\{\bar{k}_j(t)\}$ as the variables. The $2J$ functions Λ_j and Ψ_j defined in the appendix contain $2J$ variables, $z(t)$, $(\ell_j(t))$, and $\{\bar{k}_j(t)\}$. Moreover, all the other variables are uniquely determined as functions of $z(t)$, $(\ell_j(t))$, and $\{\bar{k}_j(t)\}$ as follows: $z(t)$, $(\ell_j(t))$, and $\{\bar{k}_j(t)\}$ at any point in time in the following procedure: $r(t)$ by (A2) $\rightarrow w(t)$ by (A3) $\rightarrow p_s(t)$ and $p_h(t)$ by (A5) $\rightarrow \bar{k}_1(t)$ by (A16) $\rightarrow \bar{y}_j(t)$ by (A5) $\rightarrow N_s(t)$ by (A9) $\rightarrow N_i(t)$ by (A12) $\rightarrow N_h(t)$ by (A10) $\rightarrow K_i(t)$, $K_h(t)$, and $K_s(t)$ by (A1) $\rightarrow K(t)$ by (A15) $\rightarrow F_q(t)$ by (2) $\rightarrow \bar{T}_j(t)$, $\hat{T}_j(t)$, $c_j(t)$, $c_{js}(t)$, $c_{jh}(t)$, and $s_j(t)$ by (12) $\rightarrow T_j(t)$ by (8).

The $2J$ differential equations (21) contain the same number of variables, $z_1(t)$, $(\ell_j(t))$, and $\{\bar{k}_j(t)\}$. As we can hardly solve explicitly the highly dimensional nonlinear differential equations, we deal with behavior of the dynamic system by simulation. We specify the values of the populations, human capital of the three groups,

efficiencies of health stock of the groups, parameters in the production functions, and depreciation rates of physical capital and health stock as

$$T_0 = 24, \bar{N}_1 = 10, \bar{N}_2 = 100, \bar{N}_3 = 200, h_1 = 5, h_2 = 3, h_3 = 1, m_{1h} = 0.7, \\ m_{2h} = 0.6, m_{3h} = 0.5, A_i = 1.3, A_s = 1.2, A_h = 1, \alpha_i = 0.33, \alpha_s = 0.31, \alpha_h = 0.38, \\ \delta_k = 0.05, \delta_{1h} = 0.02, \delta_{2h} = 0.025, \delta_{3h} = 0.03$$

We take on the values the parameters in the Cobb-Douglas productions approximately equal to 0.3. The specified values are often utilized in economic literature (for instance, Miles and Scott, 2005; Abel *et al.* 2007). The human capital levels of the three sectors are correspondingly specified highly in order from group 1 to group 2 and to group 3. Depreciation rates of physical capital is often fixed around 0.05 in economic studies. We follow this traditional practice. We rank the levels of human capital and utilization efficiencies of health stock highly in order from group 1 to group 2 and to group 3. The depreciation rates of health stock are specified lowly in order from group 1 to group 2 and to group 3. We specify the parameters for health caring and household preferences as follows

$$\bar{A}_1 = 1, \bar{A}_2 = 0.9, \bar{A}_3 = 0.8, \bar{\alpha}_1 = 0.35, \bar{\beta}_1 = 0.35, \bar{\alpha}_2 = 0.33, \bar{\beta}_2 = 0.33, \\ \bar{\alpha}_3 = 0.31, \bar{\beta}_3 = 0.31, \sigma_{10} = 0.4, \xi_{10} = 0.1, \gamma_{10} = 0.05, \psi_{10} = 0.025, \lambda_{10} = 0.7, \\ \sigma_{20} = 0.35, \xi_{20} = 0.1, \gamma_{20} = 0.05, \psi_{20} = 0.022, \lambda_{20} = 0.65, \sigma_{30} = 0.3, \\ \xi_{30} = 0.1, \gamma_{30} = 0.05, \psi_{30} = 0.02, \lambda_{30} = 0.6,$$

The requirement $\bar{\alpha}_j + \bar{\beta}_j < 1$ means decreasing returns to scale in health caring (e.g., Forster. 1989; Ehrlich and Chuma, 1990; Johansson and Löfgren, 1995; and van Zon and Muysken, 2001). Group 1 has the highest propensity to save and the highest propensity to take care of health. We specify the parameters in the equations for changes in health stocks as follows

$$v_{1h} = 0.7, a_{1h} = 0.4, \bar{v}_{1c} = 0.3, a_{1c} = 0.2, a_{1s} = 0.2, a_{1T} = 0.2, \pi_{1h} = 0.3, \\ \pi_{1c} = 0.4, v_{2h} = 0.7, a_{2h} = 0.35, \bar{v}_{2c} = 0.25, a_{2c} = 0.15, a_{2s} = 0.2, a_{2T} = 0.2, \\ \pi_{2h} = 0.35, \pi_{2c} = 0.45, v_{3h} = 0.7, a_{3h} = 0.3, \bar{v}_{3c} = 0.2, a_{3c} = 0.15, a_{3s} = 0.2, \\ a_{3T} = 0.2, \pi_{3h} = 0.4, \pi_{3c} = 0.5.$$

The conditions $\pi_{jc} > 0$ and $\pi_{jh} > 0.5$ mean decreasing returns to scale in health stock accumulation. The initial conditions are

$$z_1(0) = 0.0525, \bar{k}_2(0) = 1360, \bar{k}_3(0) = 250, \ell_1(0) = 76, \ell_2(0) = 28, \ell_3(0) = 15.$$

The simulation result is plotted in Figure 1. With regard to the initial conditions, most of the variables are slightly increased over time.

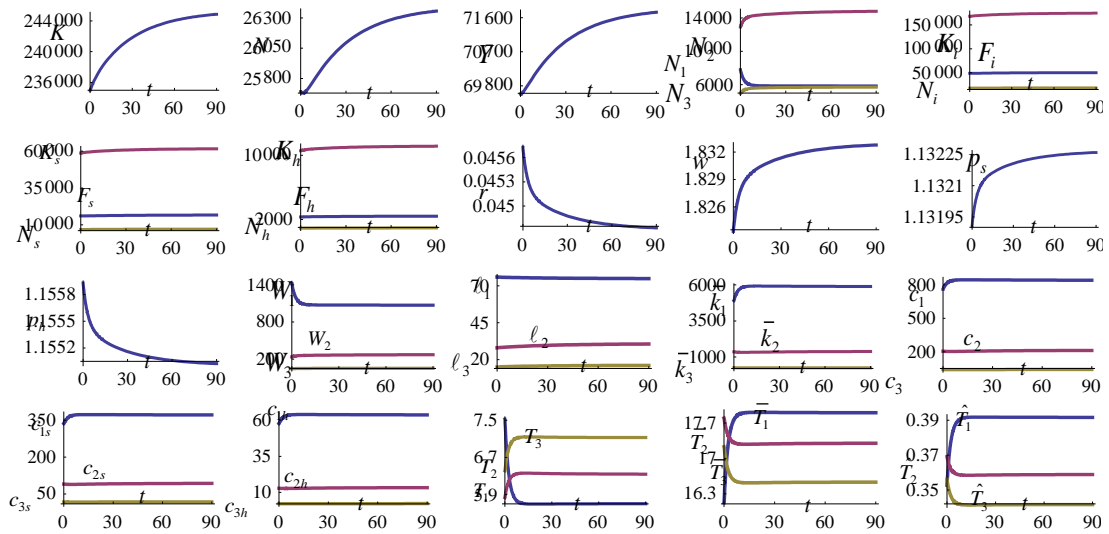


Figure 1. The Motion of the Economic System

The simulation confirms the existence of an equilibrium point. We provide the equilibrium values of the macroeconomic variables as follows

$$K = 245334, N = 26399, Y = 71866, N_1 = 5845, N_2 = 14844, N_3 = 5709.8, \\ \bar{K}_1 = 58840, \bar{K}_2 = 137956, \bar{K}_3 = 48538.$$

The economic structure of the national economy is given by

$$K_i = 175017, K_s = 61732, K_h = 11146, N_i = 18360, N_s = 7100, N_h = 939.6, \\ F_i = 50229, F_s = 16656.5, F_h = 2405.$$

We list the equilibrium values of rate of interest, wage rate, wages incomes, prices and household's behavior variables as follows

$$r = 0.045, w = 1.833, W_1 = 1071, W_2 = 2762, W_3 = 52, p_s = 1.32, p_h = 1.16, \ell_1 = 74.8, \\ \ell_2 = 30.7, \ell_3 = 16, \bar{k}_{16} = 5884, \bar{k}_{26} = 1380, \bar{k}_{36} = 243, c_1 = 840.6, c_2 = 212, c_3 = 40.1, \\ c_{1s} = 371.2, c_{2s} = 93.7, c_{3s} = 17.9, c_{1h} = 63.7, c_{2h} = 13.3, c_{3h} = 2.17, T_1 = 5.7, \\ T_2 = 6.34, T_3 = 7.13, \bar{T}_1 = 17.9, \bar{T}_2 = 17.3, \bar{T}_3 = 16.5, \hat{T}_1 = 0.39, \hat{T}_2 = 0.36, \hat{T}_3 = 0.34.$$

With the procedure in the lemma and the equilibrium values, we calculate the eigenvalues as follows:

$$\{-0.425, -0.415, -0.255, -0.039, -0.031, -0.022\}.$$

The six eigenvalues are negative. Local stability is guaranteed. Accordingly, the dynamic system always will converge to its equilibrium point if it is not far from the equilibrium. This shows that we can effectively conduct comparative dynamic analysis.

IV. TRANSITORY AND LONG-TERM EFFECTS BY COMPARATIVE DYNAMIC ANALYSIS

The lemma gives a computational procedure to describe the movement of the system. This means that we can analyze impact of changes in any exogenous conditions on the movement of the heterogeneous-household

economy. We introduce a variable $\bar{\Delta}x(t)$ to stand for the change rate of the variable, $x(t)$, in percentage due to changes in the parameter value.

4.1. Group 3’s human capital is enhanced

We now examine impact of a rise in a group’s human capital on the transitory process and long-term equilibrium point. We make an exogenous change in group 3’s human capital as follows: $h_3 : 1 \Rightarrow 1.1$. The national labor supply, national capital and national output are all enhanced. The rate of interest is augmented and the wage rate is lowered. Group 3’s wage income is greatly enhanced, while the other two groups’ wage incomes are slightly changed in the long term. From Figure 2 we see that group 3’s per household wealth, consumption levels of two goods and health are all increased, while the other two groups’ corresponding variables are slightly affected in the long term. The time distributions of all the groups are slightly changed. All the production scales are expanded. The prices of consumer good and health services slightly change. It should be noted that in order to examine how each variable changes over time in association with changes in other variables over time, we have to explain complicated interdependence of all the variables as they are closely related to each other. We simply state some effects on variables rather than detailed interdependent processes.

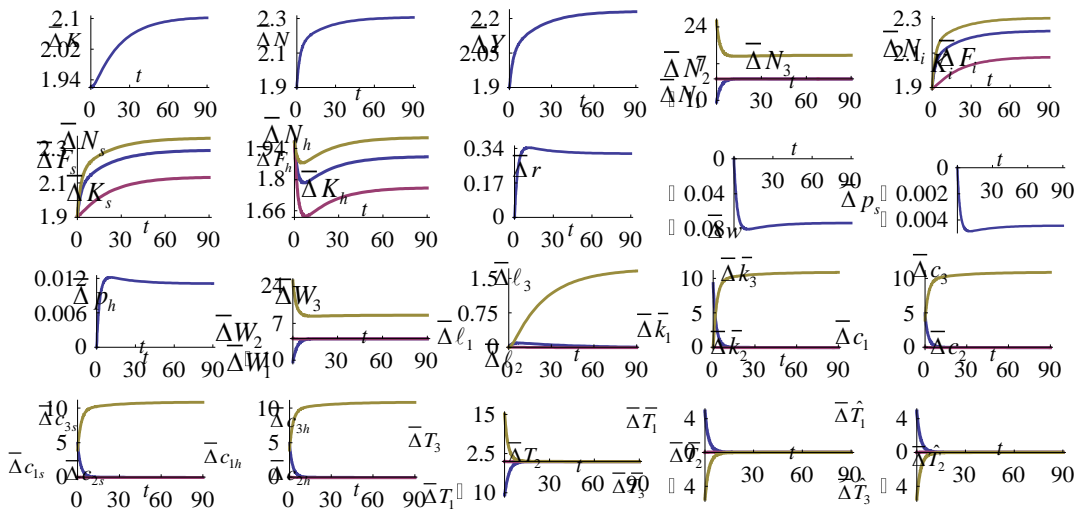


Figure 2. Group 3’s Human Capital is Enhanced

4.2. Higher efficiencies in applying health stocks

We now study a case that household apply health stocks more effectively as follows

$$m_{1h} : 0.7 \Rightarrow 0.71, m_{2h} : 0.6 \Rightarrow 0.61, m_{3h} : 0.5 \Rightarrow 0.51.$$

The result is plotted in Figure 3. The real macroeconomic variables are all improved. The national income, national wealth, and national labor supply are all increased. The populations have better health and their wage incomes are increased. There are economic structural changes. The production scales of the three sectors are expanded. The changes in the utilization effects of health stocks have slight impact on the time distributions in the long term, even though transitory processes to the long-term time distributions are affected. The households’ consumption levels of capital good and consumer good, health services, wealth and health stock are all enhanced.

$$\bar{\Delta}N_3 \qquad \bar{\Delta}F_i$$

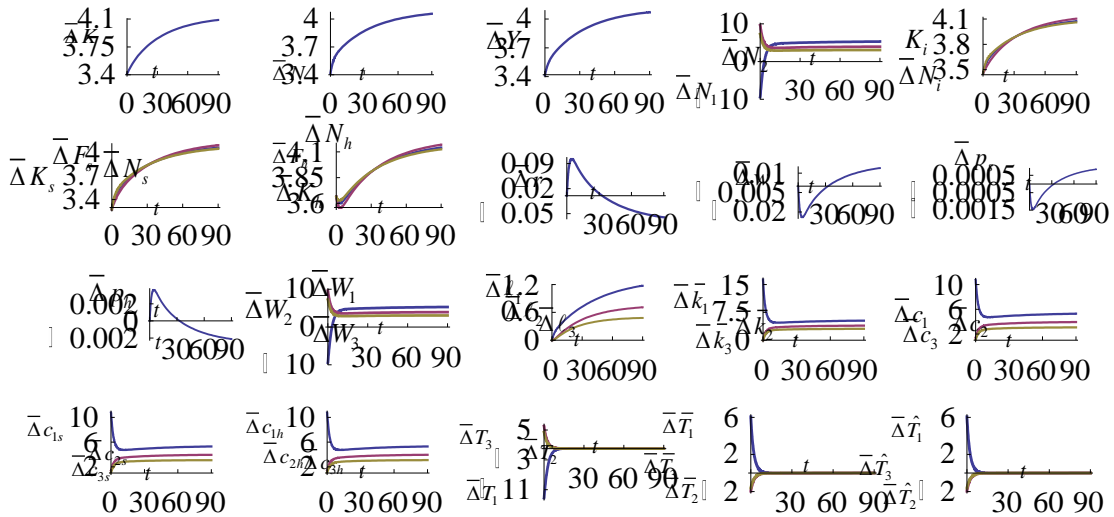


Figure 3. Higher Efficiencies in Applying Health Stocks

4.3. Stronger propensities for consuming health caring

Some empirical studies show that health has positive impact on economic growth (Bloom and Canning, 2003). Nevertheless, a possible negative impact of stronger propensity for health caring on economic growth is pointed out by van Zon and Muysken (2001): “a slow down in growth may be explained by a preference for health that is positively influenced by a growing income per head, or by an ageing population. Growth may virtually disappear for countries with high rates of decay of health, low productivity of the health-sector, or high rates of discount.” We now examine to know how changes in preferences for health caring affect economic growth in transitory processes and long-term development. As our analytical framework is a dynamic general equilibrium growth model with endogenous wealth and health stocks, we can address this kind of issues easily. We study a case that all the households increase their propensities to consume health caring as follows:

$$\psi_{10} : 0.024 \Rightarrow 0.028, \psi_{20} : 0.022 \Rightarrow 0.026, \psi_{30} : 0.02 \Rightarrow 0.024.$$

The simulation result is plotted in Figure 4. The national wealth falls in the transitory process as well as in the long term. The national labor supply and national income fall initially and rise in the long term. Hence, stronger propensities for health caring harm economic growth in the short term. In the long term national output is increased in association with falling wealth. Group 1’s labor supply falls initially and rise in the long term. The other two groups’ labor supplies rise. The capital good and consumer good sectors reduce output levels and two input factors. The health sector increases output and two input factors. The price of consumer good falls, while the price of health service rises. All the households have better health. All the households have lower wealth and consumer less capital good and consumer good. They all consumer more health services. In the long term all the households spend more hours on work and health caring and less hours on leisure.

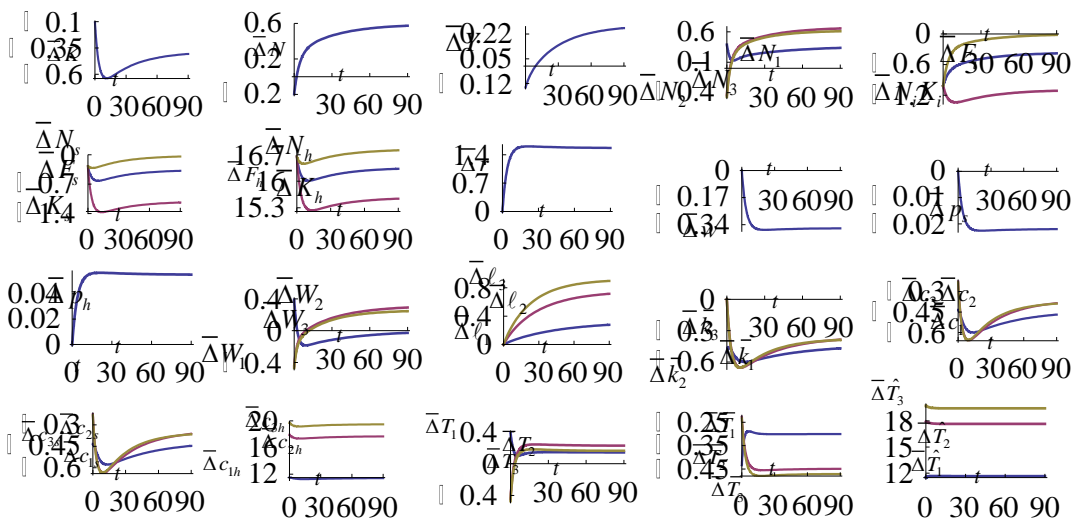


Figure 4. Stronger Propensities for Consuming Health Caring

4.4. More rich and less poor households

We fix the population components. It is important to examine what happen to the economy system if the population structure in terms of human capital and preferences varies. For simplicity of illustration, we consider an extreme case that as soon as people are classified into group, they have the same health and economic conditions as in the new group. We examine a case that the population distribution is redistributed as follows:

$$\bar{N}_1 : 10 \Rightarrow 11, \bar{N}_3 : 300 \Rightarrow 299.$$

Group 1 (richer) have more people, while group 3 (poorer) have less people. The simulation result is plotted in Figure 5. The national labor supply, national wealth and national income are all enhanced. All the production sectors are expanded. The rate of interest falls and the wage rate rises. The price of consumer good rises, while the price of health service falls. In the long term all the households receive more wage income, have more wealth, become healthier, consume more consumer good and capital good.

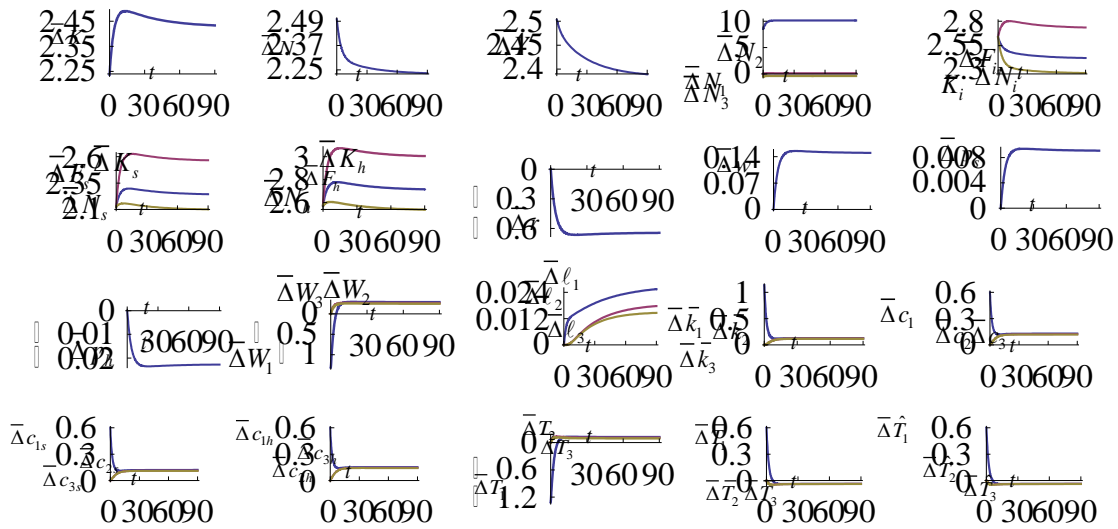


Figure 5. International Migration to Country 1

4.5. Stronger decreasing returns to scale in health caring

We now study the impact that health caring exhibits more decreasing returns to scale as follows

$$\pi_{1h} : 0.3 \Rightarrow 0.32, \pi_{2h} : 0.35 \Rightarrow 0.37, \pi_{3h} : 0.4 \Rightarrow 0.42.$$

The simulation result is plotted in Figure 6. The national wealth, national income and national labor supply are all reduced. In the long term people slightly change their time distributions. Health conditions of all the groups are deteriorated. People have less wealth and consume less consumer good and capital good. They have lower consumption levels of health service.

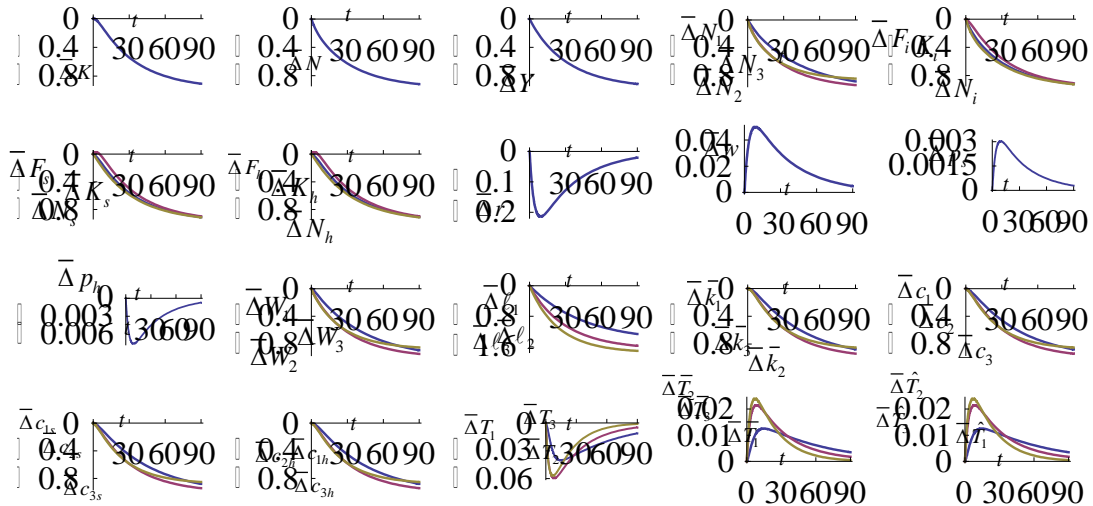


Figure 6. Stronger Decreasing Returns to Scale in Health Caring

V. CONCLUDING REMARKS

This paper proposed a dynamic general equilibrium model with endogenous wealth and health.

The economy is composed of any number of groups of people. The economy is composed of three economic sectors, capital good, consumer good, and health sectors. We described the economic structure and production technologies on the basis of Walrasian general equilibrium theory and neoclassical growth theory. The utility function proposed by Zhang was applied to describe the behavior of households. In our approach wealth and income inequalities between households are caused by heterogeneity in households' preferences, health and human capital levels. The three sectors use capital and labor as inputs. Markets are perfectly competitive. Factor markets work well and available input factors are fully utilized at every moment. Wealth accumulation is through saving and change in health stock is through health caring and consuming goods and services. We first built the dynamic general equilibrium model. Then we provided a computational procedure so that we can easily follow movement of the economic system with specified parameter values and proper initial conditions. For illustration we simulated the heterogeneous-household model with three types of households. We identified the existence of a locally stable equilibrium point for the given parameters. We plotted the motion of the economy and carried out comparative dynamic analysis with regard to changes in some parameters. We may further develop the model in some directions. For instance, it is reasonable to examine economic dynamics when utility functions or production functions are taken on other functional forms. Government intervention in health caring, taxation, and national debts due to public health expenditures are important issues (e.g., Cremer et al., 2012). Issues related to health insurance are significant for understanding health dynamics (e.g., Zhao, 2017).

Appendix: Proving the Lemma

From (3) we have

$$z \equiv \frac{r + \delta_j}{w_j} = \frac{N_q}{\bar{\beta}_q K_q}, \quad q = i, s, h, \tag{A1}$$

where $\bar{\beta}_q \equiv \beta_q / \alpha_q$. Equations (A1) and (3) imply

$$r(z) = \alpha z^{\beta_i} - \delta_k, \tag{A2}$$

where $\alpha \equiv \alpha_i A_i \bar{\beta}_i^{\beta_i}$. Solve (A1)

$$w(z) = \frac{r + \delta_k}{z}. \tag{A3}$$

Equations (4) imply

$$p_q(z) = \frac{\bar{\beta}_q^{\alpha_q} w z^{\alpha_q}}{\beta_q A_q}, \quad q = s, h. \tag{A4}$$

From (9) we have

$$\bar{y}_j = R \bar{k}_j + \bar{W}_j, \tag{A5}$$

where $\bar{W}_j \equiv h_j \ell_j^{m_j} T_0 w$. Substituting (12) into (8) yields

$$T_j = T_0 - \left(\frac{\sigma_j}{\bar{p}_j} + \frac{\hat{\psi}_j}{\bar{p}_h} \right) \bar{y}_j. \tag{A6}$$

Substitute (A5) into (A6)

$$T_j = r_j - R_j \bar{k}_j, \tag{A7}$$

where

$$r_j \equiv T_0 - (\sigma_j + \hat{\psi}_j) \frac{\bar{W}_j}{\bar{p}_j}, \quad R_j \equiv (\sigma_j + \hat{\psi}_j) \frac{R}{\bar{p}_j}.$$

From (A7) and (1) we get

$$N = \bar{r} - \bar{R}_1 \bar{k}_1, \tag{A8}$$

where

$$\bar{r} \equiv \sum_{j=1}^J h_j \ell_j^{m_j} \bar{N}_j r_j - \sum_{j=2}^J \bar{R}_j \bar{k}_j, \quad \bar{R}_j \equiv h_j \ell_j^{m_j} \bar{N}_j R_j.$$

Insert $p_s c_{js} = \gamma_j \bar{y}_j$ in (15)

$$N_s = \frac{1}{p_s f_s} \sum_{j=1}^J \gamma_j \bar{y}_j \bar{N}_j, \tag{A9}$$

where

$$f_s \equiv \frac{F_s}{N_s} = \frac{A_s}{(\bar{\beta}_s z)^{\alpha_s}}.$$

Insert $p_{jh} c_{jh} = \psi_j \bar{y}_j$ in (16)

$$N_h = \frac{1}{p_h f_h} \sum_{j=1}^J \psi_j \bar{y}_j \bar{N}_j, \tag{A10}$$

where we also use

$$f_h \equiv \frac{F_h}{N_h} = \frac{A_h}{(\bar{\beta}_h z)^{\alpha_h}}.$$

Insert (A8) - (A10) in (20)

$$N_i = \tilde{r} - \bar{R}_1 \bar{k}_1 - \tilde{r}_0 \bar{y}_1, \tag{A11}$$

where

$$\tilde{r} \equiv \bar{r} - \frac{1}{p_h f_h} \sum_{j=2}^J \psi_j \bar{y}_j \bar{N}_j - \frac{1}{p_s f_s} \sum_{j=2}^J \gamma_j \bar{y}_j \bar{N}_j, \quad \tilde{r}_0 \equiv \left(\frac{\psi_1}{p_h f_h} + \frac{\gamma_1}{p_s f_s} \right) \bar{N}_1.$$

Insert (A5) in (A12)

$$N_i = \tilde{r} - \tilde{r}_0 \bar{W}_1 - (\bar{R}_1 + \tilde{r}_0 R) \bar{k}_1. \tag{A12}$$

From (A1) in (18) we have

$$K = \left(\frac{N_i}{\beta_i} + \frac{N_h}{\beta_h} + \frac{N_s}{\beta_s} \right) \frac{1}{z}. \tag{A13}$$

Insert (A12), (A9) and (A10) into (A13)

$$K = \bar{n} + \bar{n}_0 \bar{y}_1 - \frac{\bar{R}_1 + \tilde{r}_0 R}{z \beta_i} \bar{k}_1, \tag{A14}$$

where

$$\bar{n} \equiv \left(\frac{\tilde{r} - \tilde{r}_0 \bar{W}_1}{\beta_i} + \frac{1}{\beta_h p_h f_h} \sum_{j=2}^J \psi_j \bar{y}_j \bar{N}_j + \frac{1}{\beta_s p_s f_s} \sum_{j=2}^J \gamma_j \bar{y}_j \bar{N}_j \right) \frac{1}{z},$$

$$\bar{n}_0 \equiv \left(\frac{\psi_1}{\beta_h p_h f_h} + \frac{\gamma_1}{\beta_s p_s f_s} \right) \frac{\bar{N}_1}{z}.$$

Insert (A5) in (A14)

$$K = \bar{n} + \bar{W}_1 + \left(\bar{n}_0 R - \frac{\bar{R}_1 + \tilde{r}_0 R}{z \beta_i} \right) \bar{k}_1. \tag{A15}$$

From (A15) and (19) we solve

$$\bar{k}_1 = \Lambda(z, \{\bar{k}_j\}, (\ell_j)) \equiv \left(\sum_{j=2}^J \bar{k}_j \bar{N}_j - \bar{n} - \bar{W}_1 \right) \left(\bar{n}_0 R - \frac{\bar{R}_1 + \tilde{r}_0 R}{z \beta_i} - \bar{N}_1 \right)^{-1}. \tag{A16}$$

We show now that all the variables are represented as functions of z , (ℓ_j) , and $\{\bar{k}_j\}$ at any point in time in the following procedure: r by (A2) $\rightarrow w$ by (A3) $\rightarrow p_s$ and p_h by (A5) $\rightarrow \bar{k}_1$ by (A16) $\rightarrow \bar{y}_j$ by (A5) $\rightarrow N_s$ by (A9) $\rightarrow N_i$ by (A12) $\rightarrow N_h$ by (A10) $\rightarrow K_i, K_h$, and K_s by (A1) $\rightarrow K$ by (A15) $\rightarrow F_q$ by (2) $\rightarrow \bar{T}_j$, \hat{T}_j , c_j , c_{js} , c_{jh} , and s_j by (12) $\rightarrow T_j$ by (8). From this procedure and (13) and (14), we have

$$\dot{\bar{k}}_1 = \Lambda_0(z, \{\bar{k}_j\}, (\ell_j)) \equiv s_1 - \bar{k}_1, \tag{A17}$$

$$\dot{\bar{k}}_j = \Lambda_j(z, \{\bar{k}_j\}, (\ell_j)) \equiv s_j - \bar{k}_j, \quad j = 2, \dots, J,$$

$$\dot{\ell}_j = \Psi_j(z, \{\bar{k}_j\}, (\ell_j)) \equiv \frac{\bar{v}_{jh} \bar{c}^{a_{jh}}}{\ell_j^{\pi_{jh}}} + \frac{\bar{v}_{jc} c_j^{a_{jc}} c_{js}^{a_{js}} \bar{T}_j^{a_{jT}}}{\ell_j^{\pi_{jc}}} - \delta_{jh} \ell_j, \quad j = 1, \dots, J. \tag{A18}$$

Take derivatives of (A16) in t

$$\dot{\bar{k}}_1 = \frac{\partial \Lambda}{\partial z_1} \dot{z}_1 + \sum_{j=2}^J \Lambda_j \frac{\partial \Lambda}{\partial \bar{k}_j} + \sum_{j=1}^J \Psi_j \frac{\partial \Lambda}{\partial \ell_j}, \tag{A19}$$

where we also use (A18). From (A17) and (A19), we solve

$$\dot{z} = \Lambda_1(z, \{\bar{k}_j\}, (\ell_j)) \equiv \left(\Lambda_0 - \sum_{j=2}^J \Lambda_j \frac{\partial \Lambda}{\partial \bar{k}_j} - \sum_{j=1}^J \Psi_j \frac{\partial \Lambda}{\partial \ell_j} \right) \left(\frac{\partial \Lambda}{\partial z_1} \right)^{-1}. \quad (\text{A20})$$

We solve the dynamic system with (A20) and (A18) and the rest variables by the procedure provided before. We thus checked the lemma.

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